Int. J. Heat Mass Transfer. Vol. 16, pp. 437–441. Pergamon Press 1973. Printed in Great Britain

MICROPOLAR FREE CONVECTION FLOW

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(*Received* 1 *March* 1972)

Abstract— The flow and heat-transfer characteristics of the free convection micropolar flow are described. It is found that decreasing rates of heat-transfer can be achieved by making the Newtonian solvent more and more micropolar. The micropolar characteristics are determined by the two non-dimensional parameters R (an index to microstructure character) and A (an index to substructure character). The effect of R is more pronounced on the flow and temperature fields while that of A is relatively less.

NOMENCLATURE

A, non-dimensional micro-rotation Parameter $\equiv \gamma/kL^2$;
F_{x_1}, body force acting in the negative direction of x_1-axis;
k_c, thermal conductivity;
$2L$, distance between the plates;
m, wall heat ratio parameter; change in m produces unequal temperatures at the walls;
M_s, micro-stresses;
Nu, Nusselt numbers;
p, pressure;
p_e, equilibrium state pressure;
P_D, the pressure difference $p - p_e$;
Q, mass flow rate $\int u\,dy$;
R, non-dimensional micro-rotation parameter k/μ;
S, non-dimensional wall heat parameter $\mu v_0^2/k_c\theta_1$;
T_1, T_2, constant wall temperatures;
T_e, equilibrium state temperature;
T_{12}, shear-stress;
u, non-dimensional velocity component;
v_1, v_2, v_3, velocity components in x_1, x_2 and x_3 directions;
x_1, x_2, x_3, Cartesian coordinates;
y, non-dimensional coordinates perpendicular to the plates;
β, coefficient of volume expansion;
ρ, density;
μ, κ, γ, viscosity coefficients;
ω, non-dimensional micro-rotation component $LS\nu_3/u_0$;
τ, non-dimensional temperature $S\theta/\theta_1$;
θ_1, temperature difference $\equiv T_1 - T_e$;
θ, temperature;
ν_1, ν_2, ν_3, micro-rotation components.

1. INTRODUCTION

EARLIER, many attempts have been made at a nonclassical approach to the problem of fluids containing a substructure. Anzelius [1] recognized that fluids with oblong molecules behaved in an unexpected manner in a shear field. As a major contribution towards the modern continuum theories for nonclassical fluid mechanics, Ericksen [2, 3] has developed field equations, which takes into account the presence of substructure in the fluid. This theory has its own restrictions towards an essentially dilute suspension situations. However, recent experiments due to Hoyt and Fabula [4], Vogel and Patterson [5], with fluids containing extremely small amount of polymeric additives, have indicated that the skin friction near a rigid body in such fluids is considerably lower than that in the same fluids without additives. As a general extension to Ericksen's theory and in support of the above experiments concerning the additives, Eringen [6] has proposed the theory of micropolar fluids. This theory fully explains the inertial characteristics of the substructure particles which are also allowed to undergo rotation and deformations.

It is of interest to apply this modern tool to a broadened class of real problems which have got abundent technological importance. In view of its importance in the fields of aeronautics, atomic power, chemical engineering and space research, a free connection problem has been given consideration. The heat effect of side walls on natural convection between parallel plates has been a popular field of research for a considerable length of time. In this situation, to a greater extent than in any other heat-transfer situations, flow behavior is intimately related with energy and mass transfer. This is so because the more interesting aspects of body force effects very often involve a coupling between momentum transport and the convective transfer of energy and mass. Several analytical studies of free convection flows in the presence of various geometries have been reported in literature. Of particular interest is the problem discussed by Ostrach [7]. He analysed the fully developed laminar natural convection flow between two vertical parallel plates maintained at different constant temperatures. The aim of the present investigation is to study Ostrach's problem, replacing the Newtonian fluid by a micropolar fluid model. We solve the coupled set of non-linear simultaneous equations governing the flow, micro-rotation and temperature fields, by applying a suitable iteration technique. We notice from the numerical computations, the difference between the zeroth and first order approximations of all the flow and heat-transfer parameter is small, indicating that the iteration procedure used is convergent and quite effective.

2. FORMULATION AND SOLUTION OF THE PROBLEM

The two infinite vertical parallel walls are oriented in the direction of x_1-axis and are parallel to the direction of the body force. The x_2-axis is in the direction perpendicular to the walls. The walls are kept at a distance $2L$ apart and maintained at constant temperatures T_1 and T_2, the fluid is supposed to be free from external couples. In view of the above conditions we take

$$\left.\begin{array}{lll} v_1 = v_1(x_2), & v_2 = 0, & v_3 = 0, \\ \nu_1 = 0, & \nu_2 = 0, & \nu_3 = v_c(x_2), \\ p = p(x_1, x_2), & T = T(x_2). & \end{array}\right\} \quad (1)$$

Using the conditions of the equilibrium (Hydrostatic) state of the fluid and assuming that the density of the fluid is a function of temperature alone, the equations of motion, continuity and energy for the micropolar flow, are obtained as follows:

$$(\mu + k)\frac{d^2 v_1}{dx_2^2} + k\frac{d\nu_3}{dx_2} - \frac{dP_D}{dx_1} + \rho\beta F_{x_1}\theta = 0, \quad (2)$$

$$\gamma\frac{d^2 \nu_3}{dx_2^2} - k\frac{dv_1}{dx_2} - 2k\nu_3 = 0, \quad (3)$$

$$k_c\frac{d^2\theta}{dx_2^2} + \left(\mu + \frac{k}{2}\right)\left(\frac{dv_1}{dx_2}\right)^2 + \frac{k}{2}\left(\frac{dv_1}{dx_2} + 2\nu_3\right)^2 + \gamma\left(\frac{d\nu_3}{dx_2}\right)^2 = 0. \quad (4)$$

All the letter symbols involved in equations (1)–(4) are defined in nomenclature.

Before we introduce the simplified non-dimensional equations, we modify the pressure term in equation (2). From equation (2) we see that dP_D/dx_1 must be a constant, that is, the pressure gradient $\partial p/\partial x_1$ in the channel differs from the hydrostatic pressure gradient only by a constant. However, the pressure difference required to accelerate the fluid from the hydrostatic state to the fully developed state, and decelerate it back to the hydrostatic state must be finite. But the channel is of infinite length, and hence the constant can only be zero, and the pressure gradient in the channel should be equal to the hydrostatic pressure gradient. In other words we may take $dP_D/dx_1 = 0$. With this modification incorporated, equations (2)–(4) will be solved subject to the boundary conditions

$$x_2 = -L: \quad v_1 = 0, \quad v_3 = 0, \quad \theta = T_1 - T_e = \theta_1,$$
$$x_2 = L: \quad v_1 = 0, \quad v_3 = 0, \quad \theta = T_2 - T_e = m\theta_1. \qquad (5)$$

In order to non-dimensionalize the governing equations, we introduce the following non-dimensional variables:

$$y = \frac{x_2}{L}, \quad u = \frac{Sv_1}{u_0}, \quad \tau = \frac{S\theta}{\theta_1},$$
$$\omega = \frac{LS}{u_0} v_3, \quad u_0 = \frac{\beta\rho F_{x_1} \theta_1 L^2}{\mu},$$
$$S = \frac{\mu u_0^2}{k_c \theta_1}. \qquad (6)$$

With the help of these non-dimensional variables; equations (2)–(5) may be written in the form:

$$(1 + R)u'' + R\,\omega' + \tau = 0, \qquad (7)$$

$$A\,\omega'' - u' - 2\,\omega = 0, \qquad (8)$$

$$\tau'' + \left(1 + \frac{R}{2}\right) u'^2 + \frac{R}{2}(u' + 2\,\omega)^2 + RA\,\omega'^2 = 0, \qquad (9)$$

subject to the boundary conditions:

$$y = -1, \quad u = 0, \quad \omega = 0, \quad \tau = S,$$
$$y = 1, \quad u = 0, \quad \omega = 0, \quad \tau = mS, \qquad (10)$$

where $R = k/\mu$ and $A = \gamma/kL^2$ are the non-dimensional microrotation parameters and a prime denotes differention with respect to y.

3. SOLUTION OF THE BOUNDARY VALUE PROBLEM

The boundary value problem defined by equations (7)–(10) is nonlinear and complicated, it is difficult to get a closed form solution. Hence, we adopt an iterative procedure which replaces this single boundary value problem by a system of simple boundary value problems, which are emenable to an easy analytical solution. We present the iterative system of equations in the following form:

$$(1 + R)\,u_n'' + R\,\omega_n' + \tau_n = 0, \qquad (11)$$

$$A\,\omega_n'' - u_n' - 2\,\omega_n = 0, \qquad (12)$$

$$\tau_n'' + \left(1 + \frac{R}{2}\right) u_{n-1}'^2 + \frac{R}{2}(u_{n-1}' + 2\,\omega_{n-1})^2 + RA\,(\omega'_{n-1})^2. \qquad (13)$$

The boundary conditions (10) become:

$$u_n(\pm 1) = 0, \quad \omega_n(\pm 1) = 0,$$
$$\tau_n(-1) = S, \quad \tau_n(1) = mS, \qquad (14)$$

where $n = 0, 1, 2, 3, 4, \ldots$ and $u'_{-1} = 0, \omega_{-1} = 0, \omega'_{-1} = 0$.

The case $n = 0$, corresponds to the situation when the effects of micro-rotation and the frictional heating on temperature are neglected, and the case $n = 1$, corresponds to the situation when these effects are taken into account to the first order of approximation. The analytical solutions for these two different cases are obtained and numerical results are computed for the flow and heat-transfer parameters of practical importance. It is lengthy to describe the analytical solutions and the usual shear-stress parameters and the Nusselt numbers of the flow and heat-transfer characteristics. So, these analytical results will not be recorded here. Instead the computed mass flow rate, shear-stress components, the micro-stresses and Nusselt numbers are presented in Tables 1–3.

4. DISCUSSION OF RESULTS

We count S as an index to the increase of wall heat. With the increase of S, we notice, the

Table 1. Mass flow rate Q, for S = 1·0

A	m \ R =	0·5	1·0	2·0	4·0	8·0
1·0	1·0	0·47656	0·35663	0·23716	0·14198	0·07875
	2·0	0·73302	0·54540	0·36049	0·21471	0·11867

R	m \ A =	0·5	1·0	2·0	4·0	8·0
1·0	1·0	0·36495	0·35663	0·35168	0·34895	0·34748
	2·0	0·55841	0·54539	0·53765	0·53339	0·53114
4·0	1·0	0·14740	0·14198	0·13885	0·13716	0·13628
	2·0	0·22299	0·21471	0·20993	0·20734	0·20592

R	A	m \ S =	0·2	0·4	0·6	0·8
1·0	1·0	1·0	0·06916	0·13940	0·21072	0·28314
		2·0	0·10415	0·21077	0·31985	0·38000
1·0	5·0	1·0	0·06761	0·13626	0·20594	0·27665
		2·0	0·10182	0·20598	0·31248	0·42132
5·0	1·0	1·0	0·02340	0·04692	0·07057	0·09435
		2·0	0·03514	0·07057	0·10629	0·14230

temperature everywhere in the fluid rises, consequently the velocity also increases. This increases the mass flow rate. While the walls are having different temperatures ($m = 2$) the fluid particles near the wall having higher temperature acquire greater velocities, this further increases the mass flow rate Q. These facts can be noticed from our numerical results presented in Table 1. Also, increase in velocity gradient at the wall with S, implies increase in skin friction. Table 2 illustrates this fact. The influence of micropolar parameters R and A on velocity and temperature fields is quite significant. It is observed that the effect of R or A on the velocity profiles is to flatten them. There is a sharp decrease in velocity profiles with an increase in R while the decrease is slow with A. Similar is the behavior of the temperature distribution. These in turn result in reduction of wall shears and rate of heat-transfer. It has been reported in literature that, when hot fluid flows down the channel, problems which arise from overheating of walls may be overcome by an injection of a coolant through the walls. Methods of decreasing rates of heat-transfer

Table 2. Values of shear–stress for various values of R or A or S

		$y = -1$			$y = +1$		
	m \ R =	0·5	2·0	8·0	0·5	2·0	8·0
$A = 1$	1	1·0455	1·0233	1·0079	−1·0455	−1·0233	−1·0079
	2	1·4334	1·3814	1·3455	−1·7730	−1·7242	−1·6903
	m \ A =	0·5	2·0	8·0	0·5	2·0	8·0
$R = 1$	1	1·0355	1·0340	1·0335	−1·0355	−1·0340	−1·0335
	2	1·4088	1·4076	1·4079	−1·7522	−1·7466	−1·7442
	m \ S =	0·2	0·6	0·8	0·2	0·6	0·8
$R = A = 1$	1	0·2014	0·6124	0·8221	−0·2014	−0·6124	−0·8221
	2	0·2692	0·8261	1·1138	−0·3371	−1·0303	−1·3865

Table 3. Values of Nusselt numbers for various values of R or A or S

$S = 1, A = 1$, $R =$	0·5	2·0	8·0	
*$\overline{Nu}_1$	−0·22649	−0·11550	−0·03902	
Nu_1	0·05911	0·21465	0·40354	
$\overline{Nu}_2$	0·22649	0·11550	0·03902	
Nu_2	0·96764	0·73824	0·58044	
$S = 1, R = 1$, $A =$	0·5	2·0	8·0	
$\overline{Nu}_1$	−0·17533	−0·16926	−0·16735	
Nu_1	0·06705	0·08210	0·08706	
$\overline{Nu}_2$	0·17533	0·16926	0·16735	
Nu_2	0·86187	0·84943	0·84572	
$R = 1, A = 1$, $S = 0·2$	0·2	0·4	0·6	0·8
$\overline{Nu}_1$	−0·03431	−0·06861	−0·10291	−0·13723
Nu_1	0·41527	0·33055	0·24583	0·16890
$\overline{Nu}_2$	0·03431	0·06861	0·10291	0·13723
Nu_2	0·57080	0·64160	0·71240	0·77800

* Bars on top indicate Nusselt numbers are for equal wall temperatures ($m = 1$).

are of immense help in combustion chambers, exhaust nozzles and porous walled flow reactors. From the results of the present investigation, we may achieve these decreasing rates of heat-transfer by making the Newtonian solvent more and more micropolar. The other aspects of this problem, from the viewpoint of the development of the theory of micropolar fluids, these fluid particles contained in a small volume element will have microrotations about the centroid of the element. In order to maintain these rotations, part of the kinetic energy possessed by the fluid must be dissipated. Thus, it is expected here, the flattening of velocity profiles hence less frictional heating, consequently a decrease of temperature. The other noticeable points of this investigation are, (i) when the walls are having same temperature heat flows from both the walls to the fluid. When the walls are having different temperatures heat flows from the wall at higher temperature into the fluid, and from the fluid to the other wall, (see Table 3), (ii) the component of micro-rotation decreases as R or A increases while increases with S or m. It is negative in the left half of the channel while positive in the other half. When the walls are having equal temperature, the micro-rotation vanishes on the central line of the channel, where as the vanishing shifts towards the wall having higher temperature, in case the walls are at different temperatures.

ACKNOWLEDGEMENT

One of the authors (MB) wish to thank Dr. Samuel H. Douglas, Chairman, Mathematics Department of Grambling College, for the facilities and help offered.

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ECOULEMENT MICROPOLAIRE PAR CONVECTION NATURELLE

Résumé—On décrit ici les caractéristiques dynamiques et thermiques de l'écoulement micropolaire par convection naturelle. On trouve que des flux thermiques décroissants peuvent être atteints en rendant le dissolvant newtonien de plus en plus micropolaire. Les caractéristiques micropolaires sont déterminées par les deux paramètres sans dimension R (en relation avec la microstructure) et A (en relation avec la substructure). L'effet de R est plus prononcé que celui de A sur les champs de vitesse et de température.

MIKROPOLARE FREIE KONVEKTION

Zusammenfassung—Die Strömungs- und Wärmeübergangseigenschaften der mikropolaren freien Konvektion werden beschrieben. Es zeigt sich, dass eine Abnahme des Wärmeübergangs erreicht werden kann, wenn man das Newtonsche Lösungsmittel mehr und mehr mikropolar macht. Die mikropolaren Eigenschaften werden bestimmt durch zwei dimensionslose Parameter, R (für den Mikrostrukturcharakter) und A (für den Substrukturcharakter). Der Einfluss von R auf Strömungs- und Temperaturfelder ist deutlich ausgeprägt, der von A ziemlich schwach.

МИКРОПОЛЯРНОЕ СВОБОДНОКОНВЕКТИВНОЕ ТЕЧЕНИЕ

Аннотация—Описываются динамические и тепловые характеристики свободноконвективного микрополярного течения. Найдено, что скорость переноса тепла может быть снижена путем достижения все большей микрополярности ньютоновского растворителя. Микрополярные характеристики определяются с помощью двух безразмерных параметров R (индекса вида микроструктуры) и A (индекса вида подструктуры). По сравнению с A влияние R на течение и температурные поля более значительное.